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Distributive Judgments Under Uncertainty: Paccioli's Game Revisited

Joachim Krueger
Brown University

Many decision biases arise from the inability to ignore past events. The coherence of decisions is also compromised by the inability to fully use information related to the future. In Paccioli's game, a stake of money goes to the first player to score a certain number of wins. When the game is prematurely interrupted, they may divide the stake according to the proportions of wins relative to rounds played. Alternatively, they may assess the probability that a player would reach the criterion number of wins first if the game were continued. The first decision rule (ratio), which is past-oriented, leads to contradictions across games. The second rule (probability), which is future-oriented, does not. In seven studies, use of the ratio rule emerges across testing methods, in games of chance and games of skill, and independently of extraneous factors (such as random responding, lack of awareness, or proneness to other past-oriented biases).

When choosing between alternative courses of action, it is rational to weigh the expected costs and benefits of each action and their respective probabilities. Past events are relevant only inasmuch as they allow assessments of these probabilities and the desirability of the outcomes. In actual behavioral decision making, however, the effects of past events often extend beyond such assessments. A few examples illustrate this. When committing resources for future action, many decision makers honor sunk costs (Arkes & Blumer, 1985). They become increasingly committed to failing projects as unrecoverable investments accumulate. Scientists and gamblers are mired in the past when their bets on future events depend on events that have already occurred (Tversky & Kahneman, 1974). Historians and other students of the past are apt to succumb to the hindsight bias. Once an event has occurred, it may seem to have been inevitable, although it appeared to be unlikely in the past (Fischhoff, 1975). The fundamental attribution error, which is the principal bias affecting causal attributions, also betrays past orientation (Gilbert & Malone, 1995). Perceivers attribute behaviors in part to the person's disposition even when situational factors (e.g., coercion by others) fully explain the behavior. As a consequence, perceivers tend to overestimate the degree to which future behavior will resemble past behavior (Gilbert, Pinel, Wilson, Blumberg, & Wheatley, 1998).

The theme uniting these biases is that people fail to ignore the past when they should, that is, when they ought to realize that future events are independent of (or only loosely tied to) past events (Allison, Mackie, & Messick, 1996). This is not to say that past orientation is always irrational. Often, the past predicts the future. To the extent that there are enduring patterns, perceivers are wise to examine them closely. The message of research on past-oriented biases is that the search for past patterns often results in overconfident or inaccurate predictions of future events, or a lack of concern with the future altogether (Gilovich, 1991).

Undue respect for the past often violates the decision makers' own best interests. By honoring sunk costs, they concede that they will pay money to do what they do not want to do. By trusting hindsight, they fail to learn as much as they could (Hoch & Loewenstein, 1989). By making retrospective attributions for chance events, their predictions easily become maladaptive. When the perceived causal factor is luck, the gambler's fallacy is a possibility. Change may seem imminent after a recent good (or bad) run of events (e.g., "I will get this job because the last five companies turned me down"). When the causal factor is a disposition, as in the fundamental attribution error, stability seems likely (e.g., "I won't be able to quit smoking because I wasn't able to in the past"). At best, past orientation leads to the underestimation of the probabilistic element in the human experience; at worst, pessimistic predictions are self-fulfilling and self-defeating, whereas optimistic predictions temporarily inflate the ego.

The dichotomy of past and future orientation is a useful framework for the study of judgment and decision making. Aside from jeopardizing the decision maker's self-interest, past orientation leads to incoherent choices, which violates the need for rational decisions to avoid "outright contradictions in the policies or thought processes leading to choice" (Dawes, 1998, p. 497). The "problem of points," first described in 1494 by Fra Luca dal Borgo (also known as "Paccioli"), presents a test case of coherence (cited in Dawes, 1988). Consider a post-Renaissance version of the problem:

> Player A and Player B repeatedly toss a die. Player A wins a round if an even number comes up; Player B wins if an odd number comes up. Both have contributed $10 to the stake, and they have agreed that the player who is the first to win 6 rounds will take the entire stake. For some reason, the game stops when Player A has won 5 rounds and Player B has won 3 rounds. How should the stake be divided?

Paccioli suggested that each player should receive a proportion of the stake equivalent to the number of rounds won relative to the

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Correspondence concerning this article should be addressed to Joachim Krueger, Department of Psychology, Walter S. Hunter Laboratory of Psychology, Box 1853, Brown University, Providence, Rhode Island 02912. Electronic mail may be sent to joachim_krueger@brown.edu.

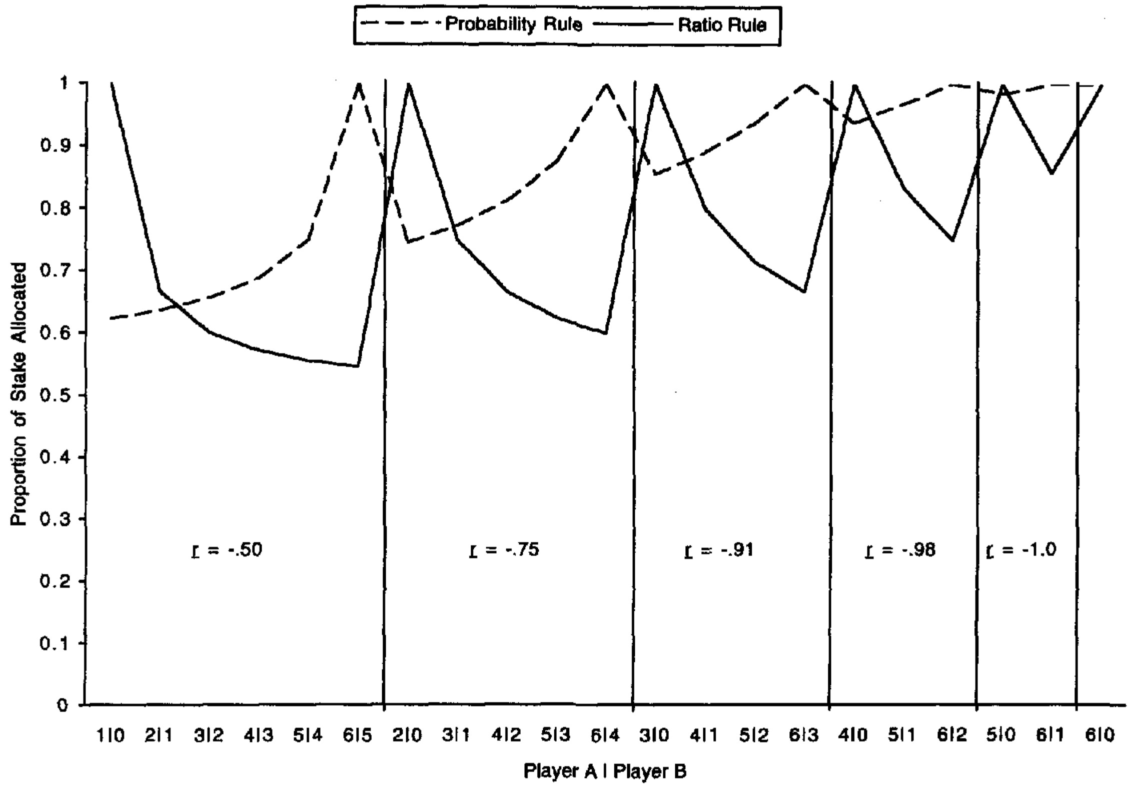

Figure 1. Allocations by the ratio rule and the probability rule for $C = 6$.

number of rounds played. In this example, Player A should get 5/8 and Player B, 3/8 of the stake. This ratio rule incorporates the first two variables of the game (Player A's and Player B's past successes) but not the third (the preset criterion C for winning the game). In other words, this rule relies on the past and ignores the future.

In this article, I examine the properties of the ratio rule and show why it leads to contradictory (i.e., irrational) decisions. As an alternative, a future-oriented probability rule is offered as a basis for rational judgment. In seven studies, I examine how well each of these rules predicts actual judgments, and I test hypotheses about why some people use the ratio rule.

The Past-Oriented Ratio Rule

According to the ratio rule, the rounds played are summed and each player receives a share of the stakes proportionate to the percentage of rounds won in the past. Consider a game with a criterion of 6 wins. Figure 1 (solid line) displays allocations to the leading Player A as a function of the difference between rounds won by Player A and rounds won by the trailing Player B. There are, for example, 6 possible outcomes in which Player A is one round ahead of Player B. The allocations for these outcomes are displayed in ascending order by the number of rounds won by Player A. If, for example, Player A has won 1 round and Player B has won none, Player A takes all.[1] If Player A has won 5 rounds and Player B has won 3 (as in the classic game), Player A takes 5/8. Player A receives less money in the classic game, although Player A has won more rounds and although Player B is further behind. If Player A reaches the criterion of 6 wins, the ratio rule grants the full amount only if Player B has won no round at all; otherwise Player A's take diminishes with the number of rounds won by Player B. This variation violates the initial agreement that the winner takes all regardless of the other player's position.[2]

The Future-Oriented Probability Rule

Peverone (1558, cited in Dawes, 1988, and Kendall, 1956) suggested that the criterion value C be incorporated in the distributive judgment.[3] A player should receive a proportion of the stake equivalent to the probability of winning if the game were played to criterion. Past wins and losses remain important as they determine each player's distance from the criterion, but this distance can only

[1] Tartaglia (1556, as cited in Kendall, 1956) realized the injustice in this.

[2] Some irrationalities can only be detected as incoherent patterns across decisions. Preference reversals predicted by prospect theory, for example, require the presentation of a decision problem in two different frames (Tversky & Kahneman, 1981). Other biases (e.g., sunk cost) are demonstrably irrational within the context of a single decision. In the present case, the judgment of irrationality follows from the observation of incoherence that the ratio rule would produce if used across games. Therefore, individual decisions by the ratio rule are suspect.

[3] Kendall (1956) noted that although Peverone had the right idea, his mathematical solution "must be one of the nearest misses in mathematics" (p. 8). Fermat and Pascal eventually solved the problem.

be covered in the future. In the classic game (Player A = 5, Player B = 3, C = 6), Player B can beat Player A only by winning 3 consecutive rounds. This is improbable ($p = .5^3 = .125$), and Player A deserves 87.50% of the stake.

The judgment for the general case is reached in two steps. First, it is determined how many possible combinations of outcomes remain until a winner is found. Second, the percentage of these combinations favoring Player A is taken as Player A's fair claim of the stake. This method will be called the *probability rule* because it involves binomial probabilities based on the assumption that Players A and B are equally likely to win any further round no matter how many rounds they have already won.

The maximum number of rounds left (N) depends on how close the players are to the criterion. N is the sum of these two distances minus 1, or $N = 2C - A - B - 1$, and the total number of remaining sequences is 2^N. Next, the number of sequences yielding a certain player as the winner is calculated. Because there are fewer opportunities for the trailing player to win, it is convenient to calculate this number as the sum of the binomial coefficients,

$$\sum_{C-B}^{N} \binom{N}{r},$$

with r ranging from C to B (the minimum number of rounds needed by Player B to reach C first) to N (the maximum number). If, for example, Player A = 4, Player B = 3, and C = 6, 5 of the 16 possible sequences yield Player B as the winner. Therefore, Player A may claim 11/16 (68.75%) of the stake.

Figure 1 (dashed line) shows that the probability rule avoids contradictions. For each difference between Players A and B, allocations to Player A increase as Player A approaches C, and they reach 1/1 when Player A reaches C. When Player A has won 1 round and Player B has won none, for example, Player A takes 62.40%; when Player A has won 6 rounds and Player B has won 5, A takes all. Across the 21 games, the allocations made by the two decision rules are modestly correlated (r = .13). The discrepancies between the two rules are more evident when the difference between Players A and B is constant. Each of the five correlations is highly negative (see Figure 1). Aside from the trivial observation that both rules allocate more to the leading player than to the trailing player, the patterns of allocations across possible games diverge. To examine which rule predicts players' intuitive decisions best, I designed the following experiments to bring out the predictive differences between the two rules.

Do Rules Rule Intuitions?

Do people's judgments agree with Paccioli's or with Peverone's? On the one hand, it is tempting to expect that players will ignore the agreed-upon criterion because it is an unrealized event in the future. Instead, they will look to what they know for sure, which is the state of the game as reflected by past wins and losses. This hypothesis is consistent with the evidence for pervasive outcome biases. On the other hand, people may be reluctant to ignore information they have. Any tendency to use numerical information available as part of the game will encourage use of the probability rule. Paccioli's game is interesting because it identifies past orientation with information neglect, whereas most previous research (e.g., on the sunk cost effect) identified past orientation with the failure to neglect irrelevant information.

The goal of the present work is to assess the extent to which players rely on the ratio rule as opposed to the probability rule. Alternative causes of past-oriented thinking are tested, and the generalizability of the findings to games of skill is examined.

Study 1: A Single Game

The main question of this initial study was whether the ratio rule or the probability rule better predicts award allocations. Participants read descriptions of Paccioli's game and judged what percentage of the stake the leading player deserved. To examine the generality of these judgments and potential signs of selfishness, players were asked to imagine either friends or strangers as their partners.

Method

Undergraduate students (N = 175) participated in small groups. Half the participants were presented with the original Paccioli game, in which Player A had gathered 5 and Player B had gathered 3 out of 6 necessary wins (Game 1). The other half were presented with a game in which Player A had won 2 and Player B had won 1 out of 3 necessary rounds (Game 2). Independently, about half the participants were identified with the leading player and the other half, with the trailing player. Last, participants were asked to imagine playing with either a friend or a stranger. After reading the description of the game, participants stated the percentage of the stake ($20.00) the leading player should claim.

Results

The data of 1 participant, who allocated nothing to the leading player, were discarded. Neither the expected outcome (winning or losing) nor the familiarity of the other player (friend or stranger) affected allocations to the leading player. Therefore, these two variables were dropped from analysis. On the average, participants allocated less than two thirds of the stake to the leading player (Ms = 66% and 62%, in Game 1 and Game 2, respectively). These averages were closer (all $ps < .001$) to the values predicted by the ratio rule (62.50% and 66.67%) than to the values predicted by the probability rule (87.50% and 75.00%).

To examine whether these averages masked individual differences in decision making, allocations were grouped in steps of 5% (≤50%, ≤55%, ≤60%, etc.) and counted. Figure 2 displays the frequencies for Game 1 (top) and Game 2 (bottom). A greater number of allocations corresponded closely to the ratio rule (Game 1: 24%; Game 2: 45%) than to the probability rule (Game 1: 3%; Game 2: 6%). The smaller variability of the allocations in Game 2 (SD = 11%) also suggested that these estimates were easier than the estimates in Game 1 (SD = 15%).

Discussion

Social factors, such as one's own relative success in the game or the identity of the partner, did not affect allocation decisions. More importantly, the ratio rule provided a better fit with percentage estimates than the probability rule did. Unexpectedly, almost 1 in 3 participants suggested an equal split. Without these preferences for equality, mean judgments would have corresponded more closely to the probability rule. Some participants may have granted equal

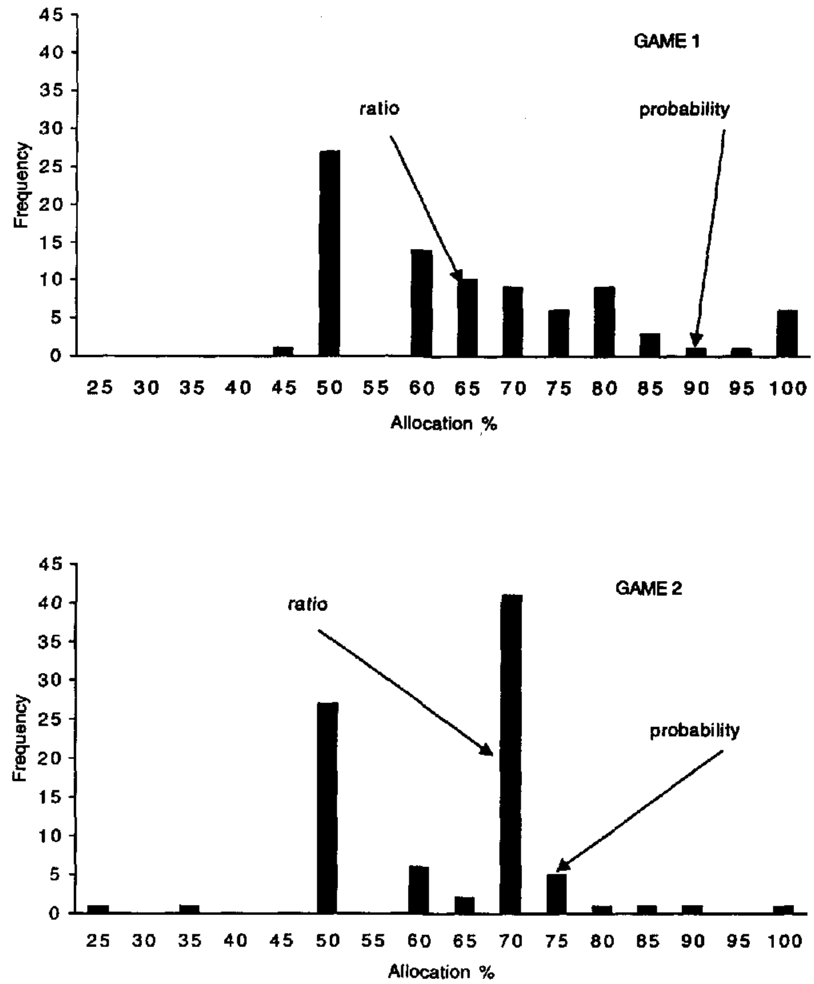

Figure 2. Frequency of allocation decisions in Game 1 (top) and Game 2 (bottom). The numbers on the abscissa are the upper bounds of the estimation intervals.

allocations to both players to heed egalitarian motives (Bazerman, White, & Loewenstein, 1995), to bypass the need to decide, or to avoid computational work. Because equal allocations are insensitive to both the history of the game (the results of past rounds) and its future (distance to criterion), the rationality of this rule depends on the consistency with which it is applied. If the equality rule is used regardless of changes in the variables of the game, it is free from contradictions. But will players use the equality rule consistently? If, for example, Player A wins 9 rounds and Player B wins 1 round out of 10 needed, Player A will be rather unhappy with an equal split, whereas Player B might feel pressured to rationalize the windfall (Kahneman & Miller, 1986). Despite its fair intent, the equality rule will not satisfy either player. If players abandon the rule when the disparity between them becomes intolerably large, their distributive judgments become incoherent. There is no metarule specifying up to which disparity equal distributions are justified.

Before the contrast between the ratio rule and the probability rule could be pursued further, it was necessary to examine whether participants recognize the limitations of the equality rule. Would they favor equality regardless of the state of the game at the time of interruption? If they abandon equality when one player is far ahead of the other, they cannot claim equality as a rule.

Study 2: The Equality Rule

Participants ($N = 70$) read descriptions of two games. In both, the criterion for winning was 10 rounds. In Game 1 (small difference), Player A had won 3 rounds, while Player B had won 1; in Game 2 (large difference), Player A had won 9 and Player B had won 3.[4] For both games, participants rated how reasonable it was to give the players their money back (equality) or to give more to the leading player A (inequality).

Method

Both games (small difference: Player A = 3, Player B = 1, $C = 10$ vs. large difference: Player A = 9, Player B = 3, $C = 10$) were worded as in

[4] The ratio was the same in the two games (3/1), and, assuming $p = .5$, Game 1 was about five times more likely to occur than Game 2. The improbability of Game 2 does not imply, however, that its distributive resolution may be different from that of the first game.

Study 1. They were printed on opposite sides of the same sheet. Half the participants read the small-difference game first and the other half read the large-difference game first. At the bottom of the sheet, they used a 7-point scale to rate the degree to which each of two strategies was a reasonable means by which to divide the \$20 (1 = *not reasonable*, 7 = *very reasonable*). One strategy was described as *equality*. Each player would take back the wagered \$10. The sheet stated that "the reasoning is that after all the game was not completed as planned." The other strategy was *inequality*, which would grant the leading player more than the trailing player. It was stated that "if the players choose this possibility, they can work out how much Player A should receive."

Results and Discussion

The findings are shown in Figure 3. A 2 (difference: small vs. large) by 2 (decision rule: equality vs. inequality) × 2 (order: first vs. second) analysis of variance (ANOVA) was performed with the last variable between participants. The incoherence in the preferences for decision rules is reflected in the interaction between the difference variable and the rule variable, $F(1, 69) = 140.23$, $p < .001$. When the difference between Players A and B was large rather than small, equality appeared less reasonable, $F(1, 69) = 95.21$, $p < .001$, and inequality appeared more reasonable, $F(1, 69) = 119.48$, $p < .001$. The order in which the two games were presented did not qualify this pattern ($F < 1$). In other words, the observed shift in decision rule was not an artifact of having the same participants evaluate both a small-difference and a high-difference game. The incoherence was the same when responses to the first game were compared between groups of participants.

Equal distributions are unfair to the leading player even when that player's advantage is small. The same player would, after all, receive a larger share if the advantage were large. Thus, equality does not solve Paccioli's problem because players would not accept it as a general rule. Because unequal allocations might as well be recognized, it is important to know by which rule players determine this inequality. The goal of the following studies was to sharpen the contrast between the ratio rule and the probability rule. In Study 3, I examined the coherence (i.e., rationality) of allocations across games to determine which rule would predict actual allocations best.

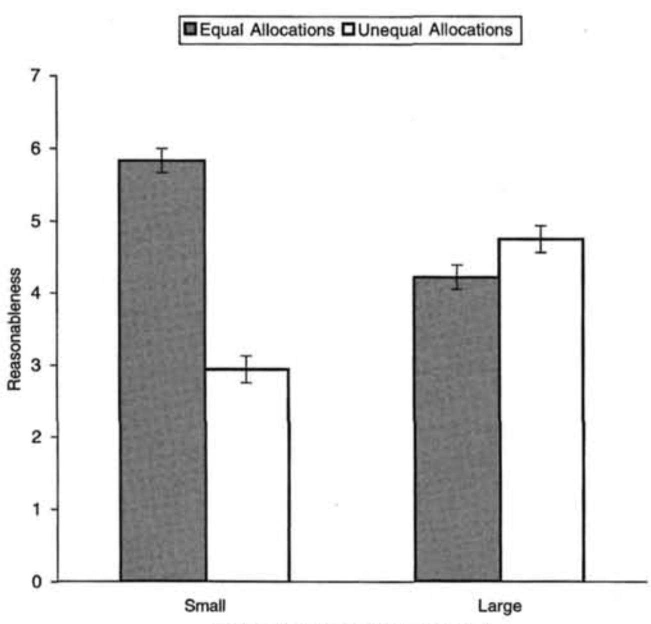

Figure 3. Average ratings of the reasonableness (with *SEMs*) of equal and unequal allocations.

Study 3: Multiple Games

Pairs of participants played four games, and each player made allocation decisions for him- or herself at the time of interruption. This method increased experimental realism and it permitted the use of correlational analyses across games.

Method

Eight experimenters recruited five pairs of participants each. Each pair played 4 games so that there were 160 games with a total of 320 allocation decisions. All players were presented with the logic of Paccioli's game. They learned that 6 wins were needed to earn the \$1 stake but that each game would be interrupted according to a predetermined schedule. Each experimenter carried a different schedule and did not reveal it to the players. Each of the 4 games was interrupted at a different point. The criterion for interruption was the number of rounds won by the leading player (2 to 5). The order of these interruptions varied so that every interruption criterion appeared in each order position equally often.

The experimenter explained the game, gave each player an answer sheet, and asked who would bet on odd numbers and who would bet on even numbers. The experimenter then rolled a die in a cup and announced the outcome. Both the players and the experimenter noted the outcome, and this process was repeated up to the preset point of interruption. Players then entered their claim to the stake (in percent) on the answer sheet. They were asked to avoid equal splits unless both players had won the same number of rounds (which never happened). The criterion ($C = 6$) was printed on each of the four response rows. When the four games were played, the experimenter debriefed the participants and gave each a token amount of \$0.50.

Results

Leaders (Player As) claimed a greater proportion of the stake for themselves ($M = 76\%$) than trailers (Player Bs) did ($M = 24\%$), $t(159) = 22.17$, $p < .001$. Because the sum of these averages was not greater than 100%, it seemed that, as in Study 1, selfishness did not distort allocations. With equal allocations absent in this study, the evidence for use of the ratio rule was stronger than in the previous study. Both averages were closer to the average ratios (Player A, $M = 75\%$; Player B, $M = 25\%$) than to the average probabilities (Player A, $M = 78\%$; Player B, $M = 22\%$).

Because ratios and probabilities were not independent ($r = .45$), allocation judgments were correlated with the predictions of each rule while the predictions of the other rule were partialed out. Among both leaders and trailers, the ratio rule predicted allocations better than the probability rule did (leaders: $rs = .56$ and $.25$, respectively, $z = 3.34$, $p < .001$; trailers: $rs = .59$ and $.04$, $z = 5.65$, $p < .001$). The point of interruption depended on the number of games the leader had won, and the state of the game then depended on how many rounds the trailer had won at that time (Games won by Player B ranged from zero to Player A's wins minus one). To determine whether the correlational analyses were affected by the differences in the frequencies of the 14 possible types of games, allocations were averaged separately for each (e.g., a game with Player A = 5 and Player B = 4 was more likely to occur than a game with Player A = 5 and Player B = 1). The partial correlations across these averages again revealed ratios as the better predictors (leaders: rs = .89 and .44, for ratios and probabilities, respectively, $z = 2.25, p < .05$; trailers: rs = .83 and .29, $z = 2.13, p < .05$).

Discussion

The mean-level analyses and the correlational analyses revealed that the ratio rule predicted allocation decisions better than the probability rule did. Especially players facing defeat reasoned almost entirely with reference to the past. This lack of future orientation was striking because, in theory, participation in different games could have stimulated future-oriented reasoning. Because the patterning of allocations across games was the central interest of this study, it remained unclear what percentage of decisions or what percentage of players could be characterized as past-oriented. The following studies addressed this issue.

Demonstrations of biases in decision making are most convincing if they are obtained under conditions most favorable for rational judgment (Krueger, 1998). Therefore, in the next three studies, I examined whether the use of the ratio rule would decrease when conditions favor the use of the future-oriented probability rule. All players were asked to adopt the perspective of the leader and to compare directly the outcomes of two interrupted games. The criterion C varied across games so that it was less likely to be ignored. Participants decided in which of the two games the leader could claim a greater proportion of the stake. Their choices would reveal the rule they used because the games were designed so that the ratio rule and the probability rule predicted opposite decisions. Moreover, decisions did not require quantitative estimates so that the complexity of the probability rule was less of a concern. I expected that even under such favorable conditions, a significant proportion of players would use the ratio rule. If so, the evidence for past orientation could be considered robust.

Because the following studies involved variations in the criterion C, it is useful to consider the effect of these variations on the relationship between the ratio rule and the probability rule. Figure 4 shows the average allocation to the leading player for all interrupted games with C ranging from 3 to 15. The first observation is that, except for very short games, the probability rule yields larger proportions for Player A than the ratio rule does and that the gap widens with increases in C. Adherence to the ratio rule not only violates the requirement of coherence but also the leader's self-interest. The second observation is that the correlation between the two rules increases with C.

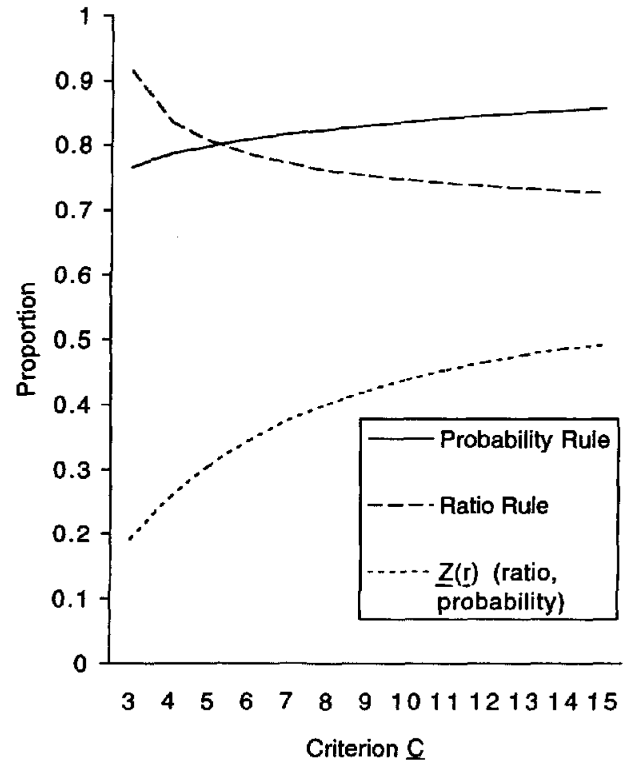

Figure 4. Allocations by the ratio rule and the probability rule for $C = 3$ to $C = 15$.

Study 4: Choice and Awareness

Each participant read the description of two games and decided in which the leader (Player A) deserved a greater share of the stake. In one game, the ratio rule granted Player A more than the probability rule did, and in the other game, the predictions were reversed. Participants needed only a general understanding of the allocation process. No precise calculations were necessary. It was also of interest whether participants knew how they made their decisions (Knight & Chao, 1991). Therefore, the two contending decision rules were briefly described, and participants indicated the one they had used.

Undergraduate students ($N = 46$) participated individually in cubicles equipped with Macintosh IIci computers. A HyperCard (1993) program controlled the presentation of the stimuli and the collection of the responses. Two games were described on the same screen with the text being essentially the same as the one presented in the introduction. In Game 1, Players A and B had won 6 and 3 rounds, respectively, when the game was interrupted, and the criterion C for winning was 15 successful rounds. The ratio rule allocated 67% of the stake to Player A, whereas the probability rule allocated 75%. In Game 2 (Player A = 3, Player B = 1, C = 13), the predictions of the two rules were inverted. Participants decided in which game Player A should claim a greater part of the stake. If they applied the probability rule, they would choose Game 1; if they applied the ratio rule, they would choose Game 2. On the next screen, participants read descriptions of the two decision rules, and with a click of the mouse they indicated which one they had used. The order of the two games was counterbalanced across participants.

The forced-choice method did not eliminate the use of the ratio rule. Many participants (33%, $z = 4.60$, $p < .001$)[5] considered Player A more deserving in Game 2. Reported choices predicted actual choices, $\chi^2(1, N = 46) = 10.58$, $p < .001$, $\Phi = .48$. Of those who used the ratio rule, 80% said they used it; of those who used the probability rule, 71% said they used it. This association between actual and reported rule use was a first indication that choice behavior in this context is not random and that preferences for either rule are stable (see also Study 7 for more relevant evidence).

The apparent use of the probability rule may have been inflated by one potential confound. Player A had won 6 rounds in the first game and 3 rounds in the second. Therefore, deciding that Player A deserved a greater share in Game 1 than in Game 2 was also consistent with a frequency rule. Participants may have felt that Player A deserved more in the first game simply because he or she had logged more wins. To test this possibility, two games were created in which the probability rule and the frequency rule made opposite predictions.

Study 5: The Frequency Rule

Participants ($N = 65$) considered a game in which the probability rule allocated 87.5% (Player A = 4, Player B = 2, $C = 5$) and a game in which it allocated 66.8% (Player A = 6, Player B = 3, $C = 11$). The frequency rule suggested that Player A deserved a greater share in Game 2. The ratio rule did not distinguish between these two games. The order of the games was varied. After choosing one of three possible allocations (larger amount in Game 1, in Game 2, or no difference), participants read descriptions of three rules (probability, frequency, and equality), and reported which one they had used.

About half the participants felt that Player A deserved more in Game 1 than in Game 2 (51% probability rule), whereas the other half (48%) felt that there was no difference. Only 1 participant felt that Player A was more deserving in Game 2. Reported choice of rule again predicted actual choice, $\chi^2(1, N = 65) = 9.67$, $p < .01$, $\Phi = .39$. Of those who used the probability rule, 70% said they used it; of those who did not use it, 71% said they did not use it. Even this design, which facilitated rational choice by juxtaposing two games, did not boost use of the probability rule to acceptably high levels. The question remains why the ratio rule is popular.

Study 6: Outcome and Skill

The next goal was to replicate the findings regarding the pervasive use of the ratio rule and to examine whether this bias can be traced to other, well-established biases of past orientation. I modified the design so that one of two players rolled the die. Two hypothetical players, Smith and Jones, played two games, and each was ahead in one game. By the probability rule, Smith was overall more deserving, whereas by the ratio rule, Jones was. Participants revealed which rule they used by choosing the player who overall deserved more.

Then they predicted which player would win if a new game were played. If participants feel that past success in a game of chance foretells future success, they exhibit an "outcome bias" (Baron & Hershey, 1988). Aside from the expected replication of the outcome bias, the question was whether users of the ratio rule are more prone to this bias than are users of the probability rule. Outcome bias may be particularly strong when a skill cue is present (Gilovich, Vallone, & Tversky, 1985). Perceptions of skill are likely when a player actively handles the apparatus generating the random events (Fleming & Darley, 1989; Rothbart & Snyder, 1970). The active player may appear to be particularly likely to win in the future if he or she was the perceived winner in the past. Again, aside from the expected replication of the skill-cue effect, the question was whether this effect is strongest among users of the ratio rule. If so, use of that rule may, in part, depend on misplaced perceptions of skill.

Method

Undergraduate students ($N = 241$) participated individually or in groups. Instructions and the parameters of the two games were presented on paper. Player A was called *Smith* in Game 1 (Player A = 6, Player B = 3, $C = 15$) and *Jones* in Game 2 (Player A = 3, Player B = 1, $C = 13$). In Game 1, Player A deserved 67% of the stake according to the ratio rule and 75% according to the probability rule. In Game 2, these predicted allocations were reversed. Thus, the ratio rule predicted that Smith's overall take should be 46% (67% in Game 1 and 100% minus 75% in Game 2), whereas Jones's take should be 54% (75% in Game 1 and 100% minus 67% in Game 2). According to the probability rule, these allocations were reversed, with Smith being overall better off.

Half the participants learned that Smith had rolled the die, whereas the other half learned that Jones had rolled the die. Participants read that "Smith took more money than Jones after Game 1, and Jones took more than Smith in Game 2. Who ended up—or should have ended up in your opinion—with more money at the final count?" After circling the name of the most deserving player, participants read the following:

> Soon, Smith and Jones will meet for another game, to be played until one of them has won 14 rounds. They will not have to cut the game short this time. Suppose you can win a quarter of their stake if you correctly predict the winner. If you're wrong, you won't lose anything. Whom do you pick?

Results

Nine participants rejected the decision problem for various reasons, ranging from "math phobia" to insistence on equal allocations. The frequencies of choosing Smith or Jones and of betting on either player are presented in Table 1.

Replication. Use of the ratio rule (i.e., considering Jones the overall winner) was reliable when Smith rolled (39%, $z = 8.88$) and when Jones rolled (26%, $z = 6.25$). The diminished use of the ratio rule when Jones himself had rolled, $\chi^2(1, N = 241) = 4.47$, $p < .05$, $h = .28$,[6] challenged the idea that mistaken perceptions of skill underlie this form of past orientation.

Outcome bias. Chance outcomes in the past affected future bets among users of the probability rule. Most of these participants (69%) felt that luck attaches itself to the recent winner, an effect not seen among users of the ratio rule (49%), $\chi^2(1, N =$

[5] The z statistic expressed the distance in standard units between the obtained proportion of "successes" (here, users of the ratio rule) and zero. With a correction for continuity (see Hays, 1978, p. 372), $z = [p - (.5/N)]/\sqrt{p(1 - p)/N}$.

[6] Cohen (1988) considers effect sizes to be small, medium, and large if $h = .20$, .50, and .80, respectively.

Table 1
Frequencies of Betting on a Smith or Jones as a Function of Decision Rule and Skill Cue: Study 6

Active (rolling) player	Smith	Jones
	Chosen rule: Ratio	
Betting on Smith	30	10
Betting on Jones	19	20
Active (rolling) player	Smith	Jones
	Chosen rule: Probability	
Betting on Smith	61	50
Betting on Jones	16	35

241) = 7.26, $p < .01$, $h = .41$. This finding contradicted the idea that use of the ratio rule arises from outcome bias.

Perceived skill. As expected, skill cues moderated outcome bias (making future bets on past winners). Most participants bet on the perceived winner if he or she rolled the die (76%) but not when the other player rolled it (51%), $\chi^2(1, N = 241) = 20.93$, $p < .001$, $h = .53$. The preferred decision rule did not moderate this effect further, $\chi^2 < 1$ (Winer, 1971). In other words, outcome bias and perceptions of skill affected bets on the future, but they were unrelated to the use of the past-oriented ratio rule.

Discussion

Use of the ratio rule did not increase when the active player was the perceived winner. This finding underscored the robustness of this past-oriented bias, and it suggested that the past-oriented bias does not arise from other, already established biases. Supporting this conclusion, users of the ratio rule were less likely than users of the probability rule to expect the perceived winner to also win in the future (outcome bias), and they were not more likely to stake their bets to irrelevant skill cues.

Before use of the past-oriented ratio rule can be regarded as a unique bias in distributive decision making, a further alternative needs to be examined. Ratios are easier to estimate than probabilities, and they minimize the experience of uncertainty. In the classic Paccioli game, for example, the ratio of rounds won over rounds played (5/8) is a certainty, whereas the probability of winning all (i.e., 7/8) is just that, a probability. The likely winner may take 7/8 of the stake, but if the game were continued, that allocation might end up appearing "undeserved" in hindsight. The ratio rule does not preclude undeserved gains, but it reduces regret. By being past-oriented, this rule does not claim that the leading player will probably win.

Possibly, some participants tried to reason probabilistically but failed. Despite efforts to estimate the future accurately, they may have overestimated the probability that the trailing player would win. If so, their decisions would have resembled the ratio rule. In their classic article on the "Belief in the Law of Small Numbers," Tversky and Kahneman (1971) described how a version of the past-oriented gambler's fallacy can contaminate future-oriented reasoning. Their thought experiment involved a sample of 50 IQs drawn from a population with a mean of 100. The first reviewed sample case was an IQ of 150. A belief in the law of small numbers would generate the expectation that the sample mean is still 100, which implies that the mean of the remaining 49 cases is 99. This expectation would mean that people fail to realize that the remaining 49 cases are a sample whose characteristics are still unknown and which cannot be influenced by the one case already reviewed. This realization, if made, would honor the assumption that the 50 observations are independent. The best estimate for the mean of the 49 is 100, and therefore, the expected total mean (including the previewed case) is 101.

I conducted an informal study to test Tversky and Kahneman's (1971) hypothesis. Students ($N = 33$) learned that the mean IQ in the population was 107. This modification recognized the fact that average IQs have risen since IQ tests were normed, and it avoided contamination of the results by tendencies to revert to round numbers (i.e., to 100). Sample size was either 10 or 50, and the previewed case had an IQ of either 57 or 157. Correct estimates for the small sample were 102 and 112 for the low and the high previewed case, respectively. For the large sample, the corresponding values were 106 and 108. Regardless of sample size and regardless of whether the previewed case was low or high, 76% of the students predicted the sample average to be the same as the population average (i.e., 107). In other words, they expected future events to correct unexpected events that occurred in the past.

If players of Paccioli's game surrender to the gambler's fallacy, they may expect the trailing player to catch up with the leader in the future. Knowing that the die is fair, they may feel that the trailer needs, deserves, and expects to win more rounds in the future than the current leader. Despite their attempt to consider future probabilities, they may not recognize the consequences of the independence of chance events.

The first goal of the final study was to test the prevalence of the ratio rule while eliminating any potential computational advantages (and thus precluding the gambler's fallacy). The second goal was to extend this test to a game whose outcome depends on skill.

Study 7: Games of Chance Versus Games of Skill

The available decision rules were paraphrased and their numerical values were presented. Participants rated each rule as to how "reasonable" it was. These judgments were collected for both a game of chance and a game of skill. Although people can probably distinguish between the two, the question was whether they would apply the ratio rule less to a game of skill than to a game of chance.

Normative considerations suggest that they do. Suppose that two players, A and B, agree to play tennis until one of them has won 6 sets.[7] Each player contributes $10 to a purse, and agrees that the winner takes all. An act of God (e.g., a thunderstorm) interrupts the game when Player A has won 5 sets and Player B has won 3 sets. How should the purse be divided? The state of the game suggests that, if it were continued, the probability of Player A's winning the next set is .625 rather than .5. In other words, the ratio obtained so far (in the past) is the best estimate of one player's skill relative to the other player. This ratio (here: 5/8) is the best estimate for the outcome of future sets. This does not mean, however, that the ratio rule is the best way to divide the purse. Instead, the ratio should be used to assess a player's probability of winning the entire game if

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[7] I thank David Budescu for suggesting this scenario.

it were played to criterion. The trailing Player B can only win the game by taking the next 3 successive sets, which means Player B's chances are 5.27% (i.e., $[3/8]^3$). Consequently, Player A's chances are 94.73%. The ratio rule penalizes the leading player more when outcomes are a matter of skill than when they are a matter of chance.

Participants received descriptions of both Paccioli's game of chance and the tennis game of skill. Three decision rules and their outcomes were described: the ratio rule, the chance probability rule ($p = .5$ for winning an individual set), and the skill probability rule ($p = .625$). The first hypothesis was that the ratio rule would be judged to be reasonable, although it no longer had a computational advantage. Preference for the ratio rule would appear to be robust if it extended to the game of skill despite the countervailing norm. The second hypothesis was that the chance probability rule would appear to be more reasonable than the skill probability rule in the game of chance, whereas the skill probability rule would appear to be more reasonable in the game of skill.

Method

Undergraduate students ($N = 63$) participated in a classroom setting. Two games with the classic parameters (Player A = 5, Player B = 3, C = 6) were presented in counterbalanced order. One was described as Paccioli's game of chance; the other was described as a tennis game. For each game, the three allocation rules were described, and their numerical outcomes were stated as odds, percentages, and dollar values (see Appendix). Using a 7-point scale, participants then rated each decision rule as to how "reasonable" it was (1 = *not reasonable*, 7 = *very reasonable*). The use of this rating scale freed participants from having to make a choice, and it permitted statistical tests with greater power.

Results and Discussion

Average ratings. Ratings of reasonableness were averaged for each condition of the 2 (game: chance vs. skill) by 2 (rule: ratio, chance probability, skill probability) × 2 (order of games) design. Means and standard deviations are displayed in Table 2. A mixed-design ANOVA was performed with repeated measures on the game and rule variables and with order as a between-subjects variable. The effect of decision rule was reliable, $F(2, 124) = 27.26, p < .001$. Simple effects analyses supported the first hypothesis, revealing that the ratio rule ($M = 5.24$) was considered more reasonable than the chance probability rule was ($M = 4.39$), $F(1, 63) = 11.21, p < .01$, which in turn was considered more reasonable than the skill probability rule was ($M = 3.44$), $F(1, 63) = 26.88, p < .001$.

Table 2
Average Ratings of "Reasonableness": Study 6

	Decision rule					
	Ratio		Chance probability		Skill probability	
Game	*M*	*SD*	*M*	*SD*	*M*	*SD*
			Game of chance first			
Chance	5.30	1.49	5.15	1.48	2.48	1.56
Skill	5.12	1.60	3.36	1.69	4.52	1.68
			Game of skill first			
Chance	5.23	1.43	4.89	1.56	3.00	1.81
Skill	5.29	1.89	4.42	1.26	3.74	1.65

Table 3
Correlations Among Ratings of Reasonableness for Three Rules and Two Games: Study 6

	Game of chance			Game of skill		
Decision rule	1	2	3	1	2	3
Game of chance						
1. Ratio	—					
2. Chance probability	−.44	—				
3. Skill probability	−.42	.11	—			
Game of skill						
1. Ratio	.46	−.26	−.14	—		
2. Chance probability	−.17	.16	.33	−.30	—	
3. Skill probability	−.41	.48	.34	−.36	−.004	—

Note. p = .05, .01, and .001 for r = .25, .33, and .41, respectively (df = 61).

The interaction between rule and game supported the second hypothesis, $F(2, 124) = 23.42, p < .001$. The chance probability rule was considered more reasonable in the game of chance ($M = 4.89$) than in the game of skill ($M = 3.88$), $F(1, 63) = 16.84, p < .001$. The skill probability rule was considered more reasonable in the game of skill ($M = 4.14$) than in the game of chance ($M = 2.73$), $F(1, 63) = 32.29, p < .001$. Remarkably, ratings for the ratio rule were nearly identical in both conditions. Participants did not seem to realize that the ratio rule is even more biased against the leading player in the game of skill than in the game of chance.

The only other reliable effect was a three-way interaction, $F(2, 124) = 8.49, p < .01$. The interaction between rule and game (see above) was reliable when the game of skill was presented first, $F(2, 64) = 23.33, p < .001$, but not when this game was presented last, $F(2, 60) = 2.84, p = .07$. Conceivably, perceptions of skill are not automatic in this context. Instead, participants seemed to realize that the skill probability rule was a reasonable distributive strategy in the tennis game only if they encountered this game after they had considered a game of chance.[8]

Correlational analyses. To examine patterns of judgment, the ratings for the three rules, as applied to the two kinds of games, were intercorrelated across participants (see Table 3). Two questions were of central interest. First, were decision rules applied consistently across games? Indeed, ratings for the ratio rule were correlated between games ($r = .46$). Participants who focused on the past in one game were likely to do so in the other. Similarly, ratings for the chance probability rule in the chance game were correlated with ratings of the skill probability rule in the skill game

[8] Categorical analyses revealed the similarities between these findings and the findings of the foregoing studies. For each respondent and condition, the decision rule with the highest rating of reasonableness was considered the preferred one. Ties were excluded. In the chance condition, the ratio rule was most popular (50%), followed by the chance probability rule (34%), and the skill probability rule (5%). In the skill condition, the corresponding percentages were 53%, 11%, and 23%.

(r = .48). Participants who recognized the optimal future-oriented rule were likely to do so in both games. Ratings of the chance probability rule in the skill game were also correlated with ratings of the skill probability rule in the chance game (r = .33), but this effect was smaller in size.[9] Also, ratings of the chance probability rule (r = .16) and ratings of the skill probability rule (r = .34) appeared to be less stable across games.

The second question was whether participants who favored one decision rule also rejected the other, although (in contrast to the previous studies) the rules were not experimentally pitted against each other. Nevertheless, participants experienced a conflict between rules. Ratings for the ratio rule were negatively correlated with ratings of both the chance probability rule (rs = −.44 and −.30 for the games of chance and skill, respectively) and with ratings of the skill probability rule (rs = −.42 and −.36 for the games of chance and skill, respectively).

General Discussion

The present studies were intended to advance an understanding of the role of time perspective in decision making. The simplicity and antiquity of Paccioli's game made it an ideal testing ground for the proposed asymmetry between past-oriented and future-oriented reasoning. The findings suggested that many players fail to reason optimally, produce incoherent judgments, and are apt to violate their own best interests. There were no social biases such as egotism or favoritism (Studies 1 and 3), nor was there consistent use of the equality rule (Study 2) or any use of the frequency rule (Study 5). Most players identified the decision rule they had used (Studies 4 and 5), and, most important, past orientation was not a by-product of other judgmental biases such as outcome bias or misperceptions of skill (Studies 6 and 7). Finally, past orientation versus future orientation appeared to be stable and mutually exclusive strategies of decision makers (Study 7).

Fundamentally, probabilistic reasoning requires a future-oriented frame of mind. When people are surprised, for example, to see that 2 of 20 party guests were born on the same day, they may ask, "What were the odds of that?" Their surprise suggests that they thought this coincidence to be unlikely a priori. The exercise of estimating or calculating the odds after the fact (which is typically done incorrectly; Diaconis & Mosteller, 1989) is merely an effort to understand or justify one's emotional response. The heuristics-and-biases paradigm, which has dominated research on statistical reasoning, assumes that people routinely estimate probabilities but that they do it poorly (Kahneman, Slovic, & Tversky, 1982). The present work suggests a deeper layer of nonnormative reasoning, namely the frequent failure to consider probabilities altogether.

My experimental strategy was to demonstrate a replicable judgmental bias and to rule out, one by one, a number of competing hypotheses about its sources. The price for this incremental progress was the repeated use of the same decision problem. Paccioli's game was recreated with only slight modifications, and the game of tennis, described in Study 7, remained structurally similar to the original scenario. In short, the asymmetry between past and future orientation was examined within a single paradigm. Extending the range of relevant reasoning problems is a task for future research.

Validity

The two decision rules are asymmetrical in that the ratio rule ignores the future (the criterion C), but the probability rule does not ignore the past (Player A's and Player B's standing at the time of interruption). This dual orientation of the probability rule is typical of rational decision rules. As noted earlier, rational choice requires an assessment of past outcomes for the calculation of future probabilities and values. One concern is that the probability rule may be entirely past-oriented because C was set in the past. But this concern begs the question of when C might be set. There is no rational way to set any particular C *after* the game has been interrupted. The leading player would be motivated to select C as the number of rounds that he or she has already won so that the allocation is 100%. The trailing player would be motivated to select an infinitely large C so that the allocation approximates 50%. To avoid this conflict, C must be established before the game begins (and it usually is, as in tennis, chess, or boxing). Before the game, both players can agree on C because neither one has an advantage yet. Future orientation then refers to the rounds still needed to reach a C that by necessity was set in the past.

A related question is whether C is necessary at all. If C remains open and if it has no maximum possible value, it may reach infinity. Thus, its expected value is infinite. The law of large numbers dictates the long-run proportion of rounds won by either player will approximate its expected value. This value is 50% in a game of chance (with p = .5), which means that allocations should be equal in Paccioli's game regardless of the state of the game at the time of interruption. As Study 2 showed, however, people do not accept equal allocations if the difference in the number of wins between Player A and Player B are large. In a game of skill, the ratios of wins by Player A and Player B approach the true ratio reflecting their differences in skill. Thus, the ratio rule might be defensible only if it is understood that the goal of the game is to assess differences in skill and if there is no criterion to be reached.

There are situations without future criteria, and therefore these situations lack a winner-takes-all goal state. When people contribute variably to a group product (e.g., raising crops, educating the young, or financing the economy), they often feel that they deserve in proportion to their efforts or investments. In these contexts, total distributable wealth increases with the sum of the investments. Thus, the ratio rule does not create the incoherence found in Paccioli's game, and indeed it is often perceived as the most equitable (Adams, 1963).

Strength

Whereas some studies (1, 3, and 7) showed widespread use of the ratio rule, others (4, 5, and 6) showed a preponderance of the probability rule. This difference in effect size can be attributed, in part, to methodological differences. Whereas the relevant analyses involved average judgments and correlations across these judgments in the first set of studies, the relevant data were probabilities of choice in the second set. Because participants could directly

[9] Specific differences between correlations were not tested for statistical significance because the statistical power for these comparisons was low. Thus, these analyses can only be considered exploratory. What should be noted, however, is the consistency of the whole pattern.

compare different games without computing numerical estimates, their task was relatively easy. Therefore, the finding that between one quarter and one third of responses corresponded to the ratio rule is robust.

When a significant proportion of decisions violates theoretical norms of rational choice, these norms fail to predict and explain human behavior. Violations of normative standards, even if committed only by a minority, are serious if there is a convincing theoretical claim that people "ought" to be able to make normative decisions (Evans, 1993). The descriptive failures of normative theories (e.g., sunk cost effects, Arkes & Blumer, 1985; outcome biases, Baron & Hershey, 1988; intransitive preferences, Tversky, 1969) have fueled the heuristics-and-biases paradigm since the early 1970s. In social perception, too, average judgments often deviate from normative standards. Many of these biases are egocentric. People overestimate how similar others are to them while feeling that they are better than most others (Krueger, 1998). The conventional group-level analyses obscure individual differences, thus masking the fact that sometimes irrationality is found only among a minority. To illustrate, Klar and Giladi (1997) concluded that "participants judged an anonymous student as better than the average student, as better than the median, and as better than most other students on a variety of desirable traits" (p. 885). Although most participants did not do this, the average rating of the anonymous person was significantly larger than the normative rating.

How small should the proportion of nonnormative responses be so that the hypothesis of rational reasoning can be retained? Following conventional practice, the present research settled for demonstrating statistically reliable departures from rational choice. The smallest of the obtained effect sizes (use of the ratio rule by about 3/10 of the participants) would not have been significant with smaller samples, but even smaller effects would have been significant with larger samples. It is interesting that most judgments concerning the statistical significance of null hypotheses are past-oriented; once the data are in (for certain), the question is how probable the data would have been if the null hypothesis had been true.

A more future-oriented approach would specify minimum required effect sizes for the phenomena under study a priori. It is difficult, however, to predict precise lower boundaries for effect sizes when theories yield only directional hypotheses. Then, the case for the robustness of a phenomenon can only be circumstantial. In the present research, various methodological devices were used to facilitate the recognition and the use of the future-oriented probability rule. Thus, the failure to render irrational responding nonsignificant was evidence for its strength.

Common to all studies reported here was reliance on respondents from an Ivy League university. These students are among the most highly selected in the country, with Scholastic Aptitude Test (SAT) scores far above average. Clearly then, the present samples are unrepresentative for the population (of adults in the United States, or even of college students). Again, however, this selection bias worked against the hypothesis of significant irrational responding. Irrationality is harder to detect among the highly intelligent. Stanovich and West (1998) reported numerous positive correlations (M = .23; see Krueger, in press) between Spearman's g factor of intelligence (mostly represented by SAT scores) and performance on various tasks related to rational reasoning. These findings suggest that preference for past-oriented reasoning should be more rather than less prevalent in the general population.

Heuristic Value

The foregoing examples of past orientation are few but representative. Most of them involve biases of commission in that irrelevant information is not ignored (Evans, 1993). Some biases can be construed as cases of anchoring and insufficient adjustment (Keysar, Barr, & Horton, 1998; Krueger, 2000; Wilson, Houston, Etling, & Brekke, 1996). According to this view, rational choice depends on controlled mental processes, whereas irrationalities do not. People encode automatically any available information; subsequently, they discount irrelevant information if they recognize it as such and if the necessary mental resources are available (Gilbert, 1998).

Paccioli's error shows that past orientation is not restricted to biases of commission. Users of the ratio rule ignored available information when they should not have. Past orientation occurred, although there were no differences between automatic and controlled processing or changes in construal. This conclusion reinforces Dawes' (1988) proposal that past orientation in its diverse incarnations is a useful organizing theme for the study of decision making. The distinction between past orientation and future orientation involves a fundamental difference in perspective. Whereas past orientation encourages the search for causal explanations and the construction of a plausible narrative, future orientation is concerned with the prediction of that which has not yet happened (Reichenbach, 1951). Both perspectives seek to reduce uncertainty, but whereas past orientation explains the meaning of what happened, future orientation predicts *what* will happen (Dawes, 1991). In past orientation, there is little uncertainty about what has happened, and yet, predictions often remain poor. Sometimes, past orientation makes predictions worse because conditions prevailing in the past may disappear in the future (Dawes, Faust, & Meehl, 1993).

Historic disciplines (e.g., evolutionary biology, archaeology, some forms of clinical psychology; Dawes, 1994) are heavily past-oriented, whereas experimental disciplines are more future-oriented (e.g., molecular biology, physics, and most forms of cognitive psychology). A final example may illustrate how the same research question, typically approached with past orientation, can yield opposite conclusions when approached with future orientation. Gould (1996) tells the evolutionary story of the horse as "life's little joke" (p. 57). The noble equine is typically portrayed as an evolutionary success, primarily because contemporary horses are larger than their eocenic ancestors. Given the prevailing idea that with time, creatures become larger and more complex, the contemporary horse appears to be a winner in the game of evolution. With past orientation, this idea is supported in a two-step analysis. First, a large and complex species (such as *equus*) is identified. Second, the history of the species is traced backward to humbler origins. By definition, this method overlooks all species and subspecies that have appeared and vanished along the way. In other words, the search for an evolutionary trend is conditioned on present outcomes (Dawes, 1993). Gould then retells the story of equine evolution from a future perspective, again in two steps. First, he identifies a species in the past (*Orohippus*). Second, he follows the history of this species forward toward its many diversifications and extinctions. From this perspective, it appears that the contemporary horse is an evolutionary loser, a remnant of a once dominant family of equines. It was saved from extinction by domestication. The joke, of course, is on the human race because its evolutionary story leads to the same contradictory conclusions depending on the perspective taken.

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(Appendix follows)

Appendix

Allocation Rules and Their Numerical Outcomes

Rule 1

Each player receives the proportion of the stake (i.e., of the $20) that corresponds to the proportion of sets won.

> Player B receives 3/8 (37.5%), which is $7.50. A receives $12.50.

Rule 2

Each player receives the proportion of the stake that corresponds to that player's chances of winning the entire game if it were played to completion. They assume that on each of the remaining sets of the game, each player is equally likely to win.

> Player B can win the entire game only by winning the next three sets in a row. The chances of that happening are $(1/2)^3 = 12.5\%$. This corresponds to $2.50. Otherwise, Player A wins. This corresponds to $17.50.

Rule 3

Each player receives the proportion of the stake that corresponds to that player's chances of winning the entire game if it were played to completion. It is assumed that on each of the remaining rounds of the game, each player's chances of winning are equivalent to the ratio of sets won so far (B = 3/8; A = 5/8).

> Player B can win the entire game only by winning the next three sets in a row. The chances of that happening are $(3/8)^3 = 5.3\%$. This corresponds to $1.05. Otherwise, Player A wins. This corresponds to $18.95.

Ratings

	Not reasonable						Very reasonable
Rule 1: ($12.50 for Player A; $7.50 for Player B)	1	2	3	4	5	6	7
Rule 2: ($17.50 for Player A; $2.50 for Player B)	1	2	3	4	5	6	7
Rule 3: ($18.95 for Player A; $1.05 for Player B)	1	2	3	4	5	6	7

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